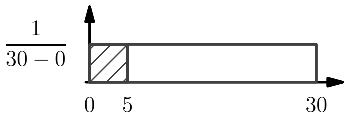
Lois de probabilités continues

▶ Exercice n°1

1. Cela revient à calculer $p(0 \leqslant X \leqslant 5) = \dfrac{5}{30} = \dfrac{1}{6}$



2. Cela revient à calculer $p(25 \leqslant X \leqslant 30) = \dfrac{5}{30} = \dfrac{1}{6}$

3. Cela revient à calculer $p(0 \leqslant X \leqslant 5) + p(25 \leqslant X \leqslant 30) = \dfrac{1}{6} + \dfrac{1}{6} = \dfrac{1}{3}$

▶ Exercice n°2

$p(X < 7) = 4 \times \dfrac{1}{a-3}$

(largeur du rectangle de 3 à 7 $=4$ et hauteur du rectangle $=\dfrac{1}{a-3}$)

$p(X < 7) = 0{,}8 \Leftrightarrow \dfrac{4}{a-3} = 0{,}8 \Leftrightarrow 4 = 0{,}8a - 2{,}4 \Leftrightarrow 6{,}4 = 0{,}8a \Leftrightarrow a = 8.$

▶ Exercice n°3

1. Cela correspond à $E(X) = \dfrac{1}{\lambda} = 40$ jours.
2. $p\,(X > 20) = 1 - p\,(0 < X < 20) = 1 - \displaystyle\int_0^{20} 0{,}025\,\mathrm{e}^{-0{,}025x}\,\mathrm{d}x$
   $= 1 - \left[-\mathrm{e}^{-0{,}025x}\right]_0^{20} = 1 - \left(-\mathrm{e}^{-0{,}5} + 1\right) = \mathrm{e}^{-0{,}5} \approx 0{,}607$

▶ Exercice n°4

1. $p\,(X < 5) = 0{,}675 \Leftrightarrow \displaystyle\int_0^{5} \lambda\,\mathrm{e}^{-\lambda x}\,\mathrm{d}x = 0{,}675 \Leftrightarrow \left[-\mathrm{e}^{-\lambda x}\right]_0^{5} = 0{,}675$
   $\Leftrightarrow -\mathrm{e}^{-5\lambda} + 1 = 0{,}675 \Leftrightarrow \mathrm{e}^{-5\lambda} = 0{,}325 \Leftrightarrow -5\lambda = \ln(0{,}325)$
   $\Leftrightarrow \lambda = \dfrac{\ln(0{,}325)}{-5} \approx 0{,}225$
2. $p\,(X < 8) = p\,(0 < X < 8) = \displaystyle\int_0^{8} 0{,}225\,\mathrm{e}^{-0{,}225x}\,\mathrm{d}x = \left[-\mathrm{e}^{-0{,}225x}\right]_0^{8}$
   $= -\mathrm{e}^{-1{,}8} + 1 \approx 0{,}835$
3. $p\,(X > 10) = 1 - p\,(0 < X < 10) = 1 - \displaystyle\int_0^{10} 0{,}225\,\mathrm{e}^{-0{,}225x}\,\mathrm{d}x$
   $= 1 - \left[-\mathrm{e}^{-0{,}225x}\right]_0^{10} = 1 - \left(-\mathrm{e}^{-2{,}25} + 1\right) = \mathrm{e}^{-2{,}25} \approx 0{,}105$

▶ Exercice n°5

1. $p\,(X \leqslant T) = 0{,}5 \Leftrightarrow \displaystyle\int_0^{T} \lambda\,\mathrm{e}^{-\lambda x}\,\mathrm{d}x = 0{,}5 \Leftrightarrow \left[-\mathrm{e}^{-\lambda x}\right]_0^{T} = 0{,}5$
   $\Leftrightarrow -\mathrm{e}^{-\lambda T} + 1 = 0{,}5 \Leftrightarrow \mathrm{e}^{-\lambda T} = 0{,}5 \Leftrightarrow -\lambda T = \ln(0{,}5)$
   $\Leftrightarrow T = \dfrac{\ln(0{,}5)}{-\lambda} = \dfrac{-\ln 2}{-\lambda} = \dfrac{\ln 2}{\lambda}$
2. $\tau = \dfrac{1}{\lambda} = \dfrac{T}{\ln 2}$